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SIM(2) and SUSY

Andrew G. Cohen

Physics Department, Boston University,
Boston, MA 02215, U.S.A.
E-mail: cohen@bu.edu

Daniel Z. Freedman

Department of Mathematics and Center for Theoretical Physics,
Massachusetts Institute of Technology
Cambridge, MA 02139, U.S.A.
E-mail: dzf@math.mit.edu

Abstract: The proposal of hep-ph/0601236, that the laws of physics in flat spacetime need be invariant only under a SIM(2) subgroup of the Lorentz group, is extended to include supersymmetry. $\mathcal{N} = 1$ SUSY gauge theories which include SIM(2) couplings for the fermions in chiral multiplets are formulated. These theories contain two conserved supercharges rather than the usual four.

Keywords: Space-Time Symmetries; Supersymmetric gauge theory.

Contents

1. Introduction

One of the present authors (A.C.), together with S. L. Glashow, has suggested [1] that the laws of physics need not be invariant under the full Lorentz group but rather under a SIM(2) subgroup, whose Lie algebra contains the four generators $T_1 = K_x + J_y$, $T_2 = K_y - J_x$, J_z, and K_z, a situation they called "Very Special Relativity" (VSR). Terms in the Lagrangian that are invariant only under this subgroup necessarily break discrete symmetries, including CP. Unlike most other subgroups, many elementary aspects of special relativity, including particle propagation, are preserved under SIM(2). The theory embodies a new mechanism for neutrino mass which conserves lepton number without introducing additional sterile states. Application to the $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ gauge symmetry of the standard model may give the electron an electric dipole moment detectable in future experiments, and lead to observable effects in the end-of-spectrum behavior in tritium β-decay. These ideas and their associated phenomenology are discussed in [1−3].

Our concern in this paper is the construction of supersymmetric field theories that are translation and SIM(2) invariant, but not Lorentz invariant. We show that the spacetime symmetry of any conventional $\mathcal{N} = 1$ SUSY gauge theory can be truncated to SIM(2) by including the characteristic SIM(2) conserving but Lorentz violating couplings for the fermions of chiral multiplets. Such theories have two conserved supercharges whose parameters are constant Majorana spinors which satisfy $\gamma_\mu n^\mu \epsilon = 0$, where $n^\mu = (1, 0, 0, 1)$ specifies the null ray fixed by the SIM(2) subgroup.[1]

[1]These theories are rather different from the theories with two supercharges proposed in [4]. Those theories require Euclidean signature, while our theories posses a null vector, and Lorentzian signature is essential.

2. SIM(2) fermions and how they propagate

In this section we review the basic fermion wave equation of SIM(2) symmetry. A free fermion is postulated in [2] to obey the equation of motion

$$\left(\partial\!\!\!/ - m^2 n\!\!\!/ \frac{1}{n \cdot \partial} - M\right) \Psi(x) = 0. \tag{2.1}$$

The term involving the fixed null vector $n^\mu \equiv (1, 0, 0, 1)$ violates Lorentz invariance, along with T, P, CT and CP-invariance. Since this null vector scales under a boost along the z-axis, SIM(2) invariance requires that it appear homogeneously. Applying the operator $\left(\partial\!\!\!/ - m^2 n\!\!\!//(n \cdot \partial) + M\right)$ and using $\{n\!\!\!/, \partial\!\!\!/\} = 2n \cdot \partial$ and $n\!\!\!/ n\!\!\!/ = 0$ leads to

$$(\Box - 2m^2 - M^2)\Psi = 0. \tag{2.2}$$

The fermion thus propagates as a massive particle with total square mass $2m^2 + M^2$. Even when $M = 0$, the plane wave solutions carry time-like 4-momentum p^μ, and the operator $1/(n \cdot \partial)$ is then well defined acting on physical wavefunctions.

There is no analogous SIM(2) modification for a free scalar field. Thus a scalar super-partner $Z(x)$ of the fermion above should satisfy

$$(\Box - 2m^2 - M^2)Z = 0. \tag{2.3}$$

3. Basic SIM(2) SUSY

We consider a chiral multiplet with Majorana spinor field $\Psi(x)$, (with chiral projections $L\Psi$, $R\Psi$), complex scalar $Z(x)$, and auxiliary field $F(x)$. The SIM(2) modification of standard SUSY is extremely simple: the kinetic action of the free chiral multiplet is the usual term $S_{\rm old}$ plus a VSR modification $S_{\rm new}$:

$$S = S_{\rm old} + S_{\rm new} \tag{3.1a}$$

$$S_{\rm old} = \int d^4x \left[-\partial^\mu \bar{Z}\partial_\mu Z + \bar{\Psi}\gamma^\mu L\partial_\mu\Psi + \bar{F}F\right] \tag{3.1b}$$

$$S_{\rm new} = -m^2 \int d^4x \left[\bar{\Psi} n\!\!\!/ \frac{1}{n \cdot \partial} L\Psi + 2\bar{Z}Z\right]. \tag{3.1c}$$

This theory is invariant under the standard SUSY transformations

$$\delta Z = \sqrt{2}\bar{\epsilon}L\Psi \qquad \delta\bar{Z} = \sqrt{2}\bar{\epsilon}R\Psi \tag{3.2a}$$

$$\delta L\Psi = -\sqrt{2}L(\partial\!\!\!/ Z + F)\epsilon \qquad \delta R\Psi = -\sqrt{2}R(\partial\!\!\!/ \bar{Z} + \bar{F})\epsilon \tag{3.2b}$$

$$\delta F = \sqrt{2}\bar{\epsilon}\,\partial\!\!\!/ L\Psi \qquad \delta\bar{F} = \sqrt{2}\bar{\epsilon}\,\partial\!\!\!/ R\Psi \tag{3.2c}$$

provided that the spinor parameters satisfy

$$n\!\!\!/ \epsilon = 0. \tag{3.3}$$

This condition decreases the number of conserved supercharges from four to two. Consistency of this condition follows from $n\!\!\!/ n\!\!\!/ = 0$.

To establish invariance we need only show that $\delta S_{\rm new}$ vanishes under the transformation rules above. The variation contains the auxiliary field term $\bar\Psi \not{n}\, 1/(n\cdot\partial) L F\epsilon$ and its conjugate. Since these terms cannot cancel with any others, we make them vanish by imposing the constraint (3.3). Four terms then remain in $\delta S_{\rm new}$, namely

$$\delta S_{\rm new} = \sqrt{2}m^2 \int d^4x \Big[-\bar\epsilon(\not{\partial}\bar Z)\not{n}\frac{1}{n\cdot\partial}L\Psi + \bar\Psi\not{n}\frac{1}{n\cdot\partial}L(\not{\partial}Z)\epsilon \\ - 2Z\bar\Psi R\epsilon - 2\bar Z\bar\epsilon L\Psi\Big]. \tag{3.4}$$

The first two terms of (3.4) can be simplified. We temporarily assume that $\epsilon(x)$ depends on x^μ to allow determination of the modified supercurrent. To start the process note that $\not{n}(\not{\partial}Z)\epsilon = 2(n\cdot\partial Z)\epsilon$ because ϵ satisfies (3.3). The first term of (3.4) then partially integrates to

$$\delta S_1 = 2\sqrt{2}m^2 \int d^4x \left[(n\cdot\partial\bar\epsilon)\bar Z\frac{1}{n\cdot\partial}L\Psi + \bar\epsilon\bar Z L\Psi\right], \tag{3.5}$$

in which the last term nicely cancels the last term of (3.4). In the second term of (3.4) we again use (3.3) to replace $\not{n}\not{\partial}Z$ with the anti-commutator, and then write $n\cdot\partial Z\,\epsilon = n\cdot\partial(Z\,\epsilon) - Zn\cdot\partial\epsilon$. The first part then nicely cancels the third term in (3.4). This leaves us with

$$\delta S_{\rm new} = 2\sqrt{2}m^2 \int d^4x \left[(n\cdot\partial\bar\epsilon)\bar Z\frac{1}{n\cdot\partial}L\Psi - \bar\Psi R\frac{1}{n\cdot\partial}(Zn\cdot\partial\epsilon)\right] \tag{3.6}$$

Since the operator $1/n\cdot\partial$ is anti-Hermitean,[2] both terms in (3.6) vanish for constant epsilon, proving that $S_{\rm new}$ is invariant under SUSY variations restricted by the constraint (3.3).

The Noether procedure tells us that the $n\cdot\partial\bar\epsilon$ and $n\cdot\partial\epsilon$ terms in (3.6) modify the supercurrent. The new current includes terms from $S_{\rm old}$ and $S_{\rm new}$:

$$\mathcal{J}^\mu = \sqrt{2}\left[L(\not{\partial}\bar Z + F)\gamma^\mu + R(\not{\partial}Z + \bar F)\gamma^\mu - 2m^2 n^\mu(L\bar Z + RZ)\frac{1}{n\cdot\partial}\right]\Psi \tag{3.7}$$

Conservation may be checked by contracting with parameters $\bar\epsilon$ which satisfy the constraint (3.3). Using the equations of motion it is then not difficult to show that $\partial_\mu\bar\epsilon\mathcal{J}^\mu = 0$. It is also straightforward to show, using plane wave expansions of the fields, that $\bar\epsilon Q = \int d^3x\,\bar\epsilon\mathcal{J}^0$ generates the conventional SUSY transformations of (3.2).

The SUSY action (3.1) is invariant under spacetime translations. The conserved energy-momentum tensor can be obtained by computing the variation of the action with a spacetime dependent translation vector $a^\mu(x)$. The stress tensor is the coefficient of $\partial_\mu a_\nu$:

$$T^{\mu\nu} = \bar\Psi\gamma^\mu\partial^\nu L\Psi - m^2\bar\Psi\overleftarrow{\frac{1}{n\cdot\partial}}\not{n}\frac{1}{n\cdot\partial}n^\mu\partial^\nu L\Psi \\ + \partial^\mu\bar Z\partial^\nu Z + \partial^\nu\bar Z\partial^\mu Z - \eta^{\mu\nu}(\partial_\rho\bar Z\partial^\rho Z + 2m^2\bar Z Z), \tag{3.8}$$

in which we have dropped terms which vanish by the fermion equation of motion. Although the fermion terms are not symmetric, this tensor is conserved on both indices. Thus a

[2] $0 = \int d^4x\, n\cdot\partial(\frac{1}{n\cdot\partial}\bar\psi\frac{1}{n\cdot\partial}\chi) = \int d^4x\,(\frac{1}{n\cdot\partial}\bar\psi\chi + \bar\psi\frac{1}{n\cdot\partial}\chi)$.

symmetric stress tensor is obtained by the simple device of adding together $\frac{1}{2}(T^{\mu\nu}+T^{\nu\mu})$. The existence of this symmetric conserved stress tensor allows us to construct conserved currents corresponding to all six conventional Lorentz generators:

$$M^{\lambda\sigma} = \int d^3x \left(x^\lambda T^{0\sigma} - x^\sigma T^{0\lambda}\right) \tag{3.9}$$

This indicates that the free theory is actually Lorentz invariant as discussed in [3]. This is as expected; the constraints of SIM(2) imply Lorentz invariant propagation for all particles. Only in the presence of interactions will Lorentz violation reveal itself, through the spin-dependent couplings of the fermion. In the interacting case the canonical stress tensor will no longer be conserved on both indices, and a symmetric tensor cannot be constructed.

Since the SIM(2) modified theory enjoys the usual transformation rules, conventional superpotential terms and couplings to the gauge multiplet $A_\mu \equiv T^a A^a_\mu$, $\lambda \equiv T^a\lambda^a$, D^a may be introduced. For example, the mass M of section 2 comes from the superpotential. The general theory is obtained by simply promoting $\partial_\mu \to D_\mu = \partial_\mu - iA_\mu$ in the formulæ above and adding the conventional terms to the action. For completeness this general SIM(2) SUSY gauge theory is given in the appendix. The only new term in the proof of invariance is

$$\delta\frac{1}{n\cdot D} = ig\frac{1}{n\cdot D}n^\mu\delta A_\mu\frac{1}{n\cdot D}, \tag{3.10}$$

but this vanishes because $n^\mu\delta A_\mu = n^\mu\bar\epsilon\gamma_\mu\lambda = 0$ by (3.3).

The SUSY algebra may be deduced by computing the SUSY transform of the supercurrent. To simplify things we drop the auxiliary field. It is important to work with conserved components of this current, so we contract with a spinor parameter constrained by (3.3) and consider $\bar\epsilon\mathcal{J}^\mu$. We then write

$$\delta_1 L\bar\epsilon_2\mathcal{J}^\mu = \sqrt{2}\bar\epsilon_2 L\left(\partial\!\!\!/\bar z\gamma^\mu - 2m^2 n^\mu\frac{1}{n\cdot\partial}\right)\delta_1 L\Psi + \cdots . \tag{3.11}$$

All bilinear boson terms are written explicitly, while the ellipses indicate the fermion terms on which we comment later. After insertion of $\delta L\Psi$ from (3.2b), and some γ-matrix algebra in the first term we get

$$\partial\!\!\!/\bar z\gamma^\mu\partial\!\!\!/ z = \tau^\mu_\nu\gamma^\nu - \gamma^{\mu\rho\sigma}\partial_\rho\bar z\partial_\sigma z, \tag{3.12}$$

with

$$\tau^\mu_\nu = \partial^\mu\bar z\partial_\nu z + \partial_\nu\bar z\partial^\mu z - \delta^\mu_\nu\partial_\rho\bar z\partial^\rho z. \tag{3.13}$$

Next we anti-symmetrize in $\epsilon_1 \leftrightarrow \epsilon_2$ to form the commutator. Using the symmetry properties of Majorana spinor bilinears, the result simplifies to

$$\delta_{[1}L\bar\epsilon_{2]}\mathcal{J}^\mu = 2\bar\epsilon_1\left[T^\mu_\nu\gamma^\nu + 2m^2\bar Z\left(\gamma^\mu Z - n^\mu\frac{1}{n\cdot\partial}\partial\!\!\!/ Z\right)\right]\epsilon_2, \tag{3.14}$$

in which we have added and subtracted the scalar mass term so that τ^μ_ν is replaced by the stress tensor T^μ_ν. The first term is what we expect, but the last term is surely not.

Fortunately it vanishes due to the spinor condition (3.3) as the following manipulations show

$$\bar{\epsilon}_1\left(\gamma^\mu Z - n^\mu \frac{1}{n\cdot\partial}\partial\!\!\!/ Z\right)\epsilon_2 = \bar{\epsilon}_1(\gamma^\mu n\cdot\partial - n^\mu \partial\!\!\!/)\frac{1}{n\cdot\partial}Z\Big)\epsilon_2$$
$$= \bar{\epsilon}_1\left(\frac{1}{2}\gamma^\mu\{n\!\!\!/, \partial\!\!\!/\} - n^\mu\partial\!\!\!/\right)\frac{1}{n\cdot\partial}Z\Big)\epsilon_2 \qquad (3.15)$$
$$= 0.$$

The final step was achieved by moving the $n\!\!\!/$ to the right or left so that it annihilates the ϵ spinors. The spinor contributions to (3.11) conform to the pattern above and need not be studied explicitly.

The net result is that the SUSY algebra appears to be conventional, namely

$$\bar{\epsilon_1}\{Q,\bar{Q}\}\epsilon_2 \ = \ 2(\bar{\epsilon}_1\gamma_\mu\epsilon_2)P^\mu, \qquad (3.16)$$

but it is important that the ϵ spinors satisfy (3.3). To incorporate this fact we note that (3.3) can be written as $\epsilon = \gamma_0\gamma_3\epsilon$, implying that the 4-vector $\bar{\epsilon}_1\gamma_\mu\epsilon_2$ is proportional to n^μ. Let $\tilde{n}^\mu$ be any vector whose scalar product with n^μ is unity, $\tilde{n}\cdot n = 1$. Then we can rewrite (3.16) as

$$\{\bar{\epsilon_1}Q, \bar{Q}\epsilon_2\} = 2\Xi_{12}\, n_\mu P^\mu \qquad (3.17)$$

where $\Xi_{12} = \tilde{n}^\nu(\bar{\epsilon}_1\gamma_\nu\epsilon_2)$. Thus only the positive definite combination $n_\mu P^\mu$ occurs in the SUSY algebra.

We now consider the determination of the vacuum state in a general theory of this type. The vacuum must minimize the scalar potential

$$V = W'(Z)\bar{W}'(\bar{Z}) + \frac{1}{2}\sum_a D^a(Z,\bar{Z})^2 + \sum 2m^2\bar{Z}Z, \qquad (3.18)$$

where the last sum includes the scalars of all chiral multiplets with SIM(2) couplings. The vacuum energy must vanish in a supersymmetric state which requires the 3 conditions

$$W'(Z) = 0, \qquad D^a(Z,\bar{Z}) = 0 \qquad \text{and} \qquad m^2 Z = 0. \qquad (3.19)$$

SUSY is spontaneously broken unless the superpotential gradient and the D-terms vanish at a point where $Z = 0$ for all multiplets with SIM(2) masses. If SUSY breaking were due only to SIM(2), the vacuum energy would be of order $V \sim m^2|\langle Z\rangle|^2$ where $\langle Z\rangle$ is a typical Vev of scalars with SIM(2) masses. For SIM(2) masses of the order of neutrino masses, $m \approx 10^{-1}$ eV, and similarly sized scalar Vevs, the vacuum energy is near the experimental value.

4. SIM(2) and the gauge multiplet

Consider a free abelian gauge multiplet with physical components A_μ, λ and standard SUSY transformation rules

$$\delta A_\mu = \bar{\epsilon}\gamma_\mu\lambda \quad \delta\lambda = \frac{1}{2}\gamma^{\rho\sigma}F_{\rho\sigma}\epsilon. \qquad (4.1)$$

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It is not hard to show that the linear equations of motion

$$\left(\not{\partial} - m^2 \not{n} \frac{1}{n \cdot \partial}\right) \lambda = 0 \tag{4.2}$$

$$\left(\partial^\mu - 2m^2 n^\mu \frac{1}{n \cdot \partial}\right) F_{\mu\nu} = 0 \tag{4.3}$$

are covariant under SUSY transformations provided that (3.3) holds. Taking the divergence of the Bianchi identity, $\partial^\mu \partial_{[\mu} F_{\rho\sigma]} = 0$, and using the modified Maxwell equation above shows that $(\Box - 2m^2) F_{\mu\nu} = 0$.

This seems like a promising start for SIM(2) SUSY in the gauge multiplet, but there are several problems.

- The SIM(2) modified Maxwell equation cannot be derived from a gauge invariant action.
- Suppose we add a current source and write

$$\left(\partial^\mu - 2m^2 n^\mu \frac{1}{n \cdot \partial}\right) F_{\mu\nu} = J_\nu. \tag{4.4}$$

  If we apply ∂^ν and use (4.4) again, we find that the current must satisfy

$$\partial^\nu J_\nu - 2m^2 n^\nu \frac{1}{n \cdot \partial} J_\nu = 0, \tag{4.5}$$

  rather than the usual conservation law.
- In the non-abelian extension of the construction above, it appears that there is no nonlinear modification of the equations of motion which obeys SUSY.

For these reasons we have not modified the action for gauge multiplets.

5. Conclusions

We have formulated Lorentz violating, but SIM(2) conserving supersymmetric field theories. The number of supersymmetries is half that required of a conventional Lorentz invariant theory. Although the supersymmetry transformations are unmodified, the absence of half the usual supercharges leads to a modified SUSY algebra involving only translations along the null direction n, rather than the full set of spacetime translations.

Although the theory is not Lorentz invariant, an old-fashioned perturbation scheme involving the Hamiltonian may be used to compute SIM(2) covariant amplitudes [3].

SIM(2) theories have novel effects that are forbidden by Lorentz invariance. The reduced number of supersymmetries in the SIM(2) context may allow for further interesting physics beyond the standard model.

Added Note: In [5] Lindström and Roĉek have obtained superspace formulations in which both the chiral and non-abelian gauge multiplet actions are modified by non-local SIM(2) terms. Their proposal overcomes all difficulties listed in section 4. In the free limit their gauge multiplet equations of motion agree with our (4.2)–(4.3).

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A. General SIM(2) SUSY gauge theories

In this appendix, we present the SIM(2) truncation of the general $\mathcal{N} = 1$ SUSY gauge theory with gauge group G. The theory contains a chiral matter multiplet $Z^\alpha, L\Psi^\alpha, F^\alpha$ in an arbitrary representation $\mathbf{R}$ of G with Hermitean generators $(T^a)^\alpha{}_\beta$. The conjugate anti-chiral multiplet $\bar{Z}_\alpha, R\Psi_\alpha, \bar{F}_\alpha$ is also required, and these matter multiplets are coupled to the gauge multiplet A^a_μ, λ^a, D^a. Representation indices are suppressed in formulas where this is unambiguous. The component fields couple through gauge and Yukawa interactions, and an optional holomorphic gauge invariant superpotential $W(z^\alpha)$.

The covariant derivatives of the various fields are

$$D_\mu\lambda^a = \partial_\mu\lambda^a + gf^{abc}A^b_\mu\lambda^c \quad \text{(A.1a)}$$

$$D_\mu Z = \partial_\mu Z - igT^a A^a_\mu Z \quad \text{(A.1b)}$$

$$D_\mu L\Psi = \partial_\mu L\Psi - igT^a A^a_\mu L\Psi \quad \text{(A.1c)}$$

$$D_\mu R\Psi = \partial_\mu R\Psi + igR\Psi T^a A^a_\mu \,. \quad \text{(A.1d)}$$

The action of the general theory is the sum of several terms $S = S_{\text{gauge}} + S_{\text{matter}} + S_{\text{coupling}} + S_F + S_{\bar{F}} + S_{\text{new}}$ where

$$S_{\text{gauge}} = \int d^4x \left[-\frac{1}{4}F^{\mu\nu a}F^a_{\mu\nu} + \frac{1}{2}\bar{\lambda}^a\gamma^\mu D_\mu\lambda^a + \frac{1}{2}D^aD^a\right] \quad \text{(A.2a)}$$

$$S_{\text{matter}} = \int d^4x \left[-D^\mu\bar{Z}D_\mu Z + \bar{\Psi}\gamma^\mu L D_\mu\Psi + \bar{F}F\right] \quad \text{(A.2b)}$$

$$S_{\text{coupling}} = g\int d^4x \left[-i\sqrt{2}(\bar{\lambda}^a\bar{Z}T^aL\Psi - \bar{\Psi}RT^aZ\lambda^a) + D^a\bar{Z}T^aZ\right] \quad \text{(A.2c)}$$

$$S_F = -\int d^4x \left[F^\alpha W_{,\alpha} - \frac{1}{2}\bar{\Psi}^\alpha L W_{,\alpha\beta}\,\Psi^\beta\right] \quad \text{(A.2d)}$$

$$S_{\bar{F}} = -\int d^4x \left[\bar{F}_\alpha\bar{W}^\alpha - \frac{1}{2}\bar{\Psi}_\alpha R\bar{W}^{,\alpha\beta}\,\Psi_\beta\right] \quad \text{(A.2e)}$$

$$S_{\text{new}} = -m^2\int d^4x \left[\bar{\Psi}\not{n}\frac{1}{n\cdot\partial}L\Psi + 2\bar{Z}Z\right] . \quad \text{(A.2f)}$$

The full action is invariant under SUSY transformation rules for the gauge multiplet

$$\delta A^a_\mu = \bar{\epsilon}\gamma_\mu\lambda^a \quad \text{(A.3a)}$$

$$\delta\lambda^a = \left[\frac{1}{2}\gamma^{\rho\sigma}F^a_{\rho\sigma} + igD^a\right]\epsilon \quad \text{(A.3b)}$$

$$\delta D^a = -\bar{\epsilon}ig\gamma^\mu D_\mu\lambda^a \,. \quad \text{(A.3c)}$$

and for the chiral and anti-chiral multiplets

$$\delta Z = \sqrt{2}\bar{\epsilon} L\Psi \qquad \delta \bar{Z} = \sqrt{2}\bar{\epsilon} R\Psi \tag{A.4a}$$

$$\delta L\Psi = -\sqrt{2} L(\gamma^\mu D_\mu Z + F)\epsilon \qquad \delta R\Psi = -\sqrt{2} R(\gamma^\mu D_\mu \bar{Z} + \bar{F})\epsilon \tag{A.4b}$$

$$\delta F = \sqrt{2}\bar{\epsilon} R(\gamma^\mu D_\mu \Psi + ig\lambda^a T^a Z) \qquad \delta \bar{F} = \sqrt{2}\bar{\epsilon} L(\gamma^\mu D_\mu \Psi - ig\lambda^a T^a Z)\,. \tag{A.4c}$$

These are the conventional transformation rules, but the spinor ϵ must satisfy $\not{n}\epsilon = 0$. The supercurrent is

$$\mathcal{J}^\mu \;=\; \gamma^{\nu\rho} F^a_{\nu\rho} \gamma^\mu \lambda^a + \sqrt{2}\left[L(\not{\partial}\bar{Z} + F)\gamma^\mu + R(\not{\partial} Z + \bar{F})\Psi - 2m^2 n^\mu (L\bar{Z} + RZ)\frac{1}{n\cdot\partial}\right]\Psi \tag{A.5}$$

where $F^\alpha = -\bar{W}_{,\alpha}$, $\bar{F}^\alpha = -W_{,\alpha}$ is the solution of the auxiliary field equation. Only the components obtained by contracting the supercurrent with spinors satisfying the constraint (3.3) are conserved.

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